Design Criteria for Multistage Microwave Amplifiers with Match Requirements at Input and Output

Giuseppe Macchiarella, Alessandro Raggi, and Elpidio Di Lorenzo

***Abstract*—An approach to the design of multistage microwave amplifiers in a prescribed frequency band with requirements about the transducer gain flatness and the maximum magnitude of the reflection coefficient at input and output is presented. The interstage equalizers are designed by imposing a suitable constraint on the maximum transducer gain, obtainable directly from the specifications; the input and output equalizer are obtained by imposing only the matching requirement. The method proposed allows a separate design of each network, which can be performed either through direct optimization or by means of a numerical synthesis.**

I. Introduction

TECHNIQUES for designing microwave amplifiers have been receiving an increasing interest in the last decade [1], [2]. The procedures generally proposed, however, allow the specifications to concern only the transducer gain in a given frequency range; some works [3], [4] have considered also the input or output matching level, but these parameters have never been taken simultaneously into account in some design procedure. As a consequence, empirical adjustments are often required in practical designs (for instance by means of CAD tools) to lower the input and output VSWR; alternatively nonreciprocal devices have to be utilized (circulators, isolators).

We discuss a novel criterion for approaching the design of multistage microwave amplifiers with specifications also on the magnitude of the input and output reflection coefficients, in addition to the transducer gain level and flatness. It is assumed that the equalizer networks are lossless and that the active devices are stable in the frequency band of the design (unconditional stability is not strictly required). For each network to be designed a constraint on a suitable parameter is derived, which allows the synthesis by means of well known procedures (direct optimization, "real frequency" technique, etc).

Section II deals with some analytical background; a simple expression for the overall transducer gain of the amplifier is derived and the approximations introduced are justified. In Sections III and IV the design approach proposed is discussed; finally in Section V an example of application of the procedure is presented.

Manuscript received April 5, 1991; revised October 22, 1992.

The authors are with the Dipartimento di Elettronica, Politecnico di Milano, 20132 Milano, Italy.

IEEE Log Number 9208360.

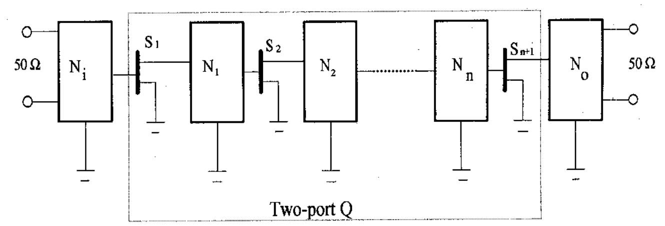


Fig. 1. General configuration of a multistage microwave amplifier.

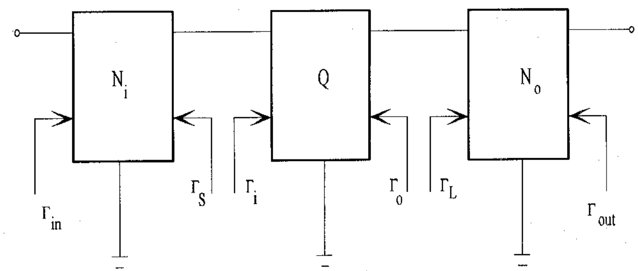


Fig. 2. Schematic representation of the multistage amplifier. Block Q contains the active devices and the interstage equalizers.

II. Analytical Backgrounds

Figure 1 shows the general configuration of a microwave multistage amplifier; as said, we assume that all the equalizing networks N_i are lossless and the active devices S_i are stable in the frequency range of the amplifier. In fact, unconditional stability is required for the overall two-port Q, which includes the active devices and the interstage networks (Fig. 2); however, severe design requirements should be more easily satisfied if very stable active devices are used (stability can be increased, at the expense of gain, by cascading the active device with a resistor).

Let be S the scattering matrix of the two-port Q; various reflection coefficients are represented in Fig. 2: in particular, the magnitude of Γ_{in} and Γ_{out} determine the level of match at the input and output of the amplifier.

Note that, being N_I and N_O lossless, the equations hold:

$$1-|\Gamma_{\text{in}}|^2=\frac{(1-|\Gamma_i|^2)(1-|\Gamma_s|^2)}{|1-\Gamma_i\Gamma_S|^2} \tag{1a}$$

$$1-|\Gamma_{\text{out}}|^2=\frac{(1-|\Gamma_o|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_o\Gamma_L|^2}. \tag{1b}$$

Moreover, Γ_i and Γ_o are given by the following well-known relationships:

$$\Gamma_i=s_{11}+\frac{s_{12}s_{21}\Gamma_L}{1-\Gamma_L s_{22}} \tag{2a}$$

$$\Gamma_o = s_{22} + \frac{s_{12}s_{21}\Gamma_S}{1-\Gamma_S s_{11}}. \tag{2b}$$

Substituting (2a) and (2b) into (1a) and (1b), the following expressions are obtained:

$$|\Gamma_{\text{in}}|^2 = \left\{1 - \frac{(1-|\Gamma_S|^2)(|1-\Gamma_L s_{22}|^2 - |s_{11}-\Delta\Gamma_L|^2)}{|1-\Gamma_L s_{22} - \Gamma_S s_{11} + \Gamma_L\Gamma_S\Delta|^2}\right\} \tag{3a}$$

$$|\Gamma_{\text{out}}|^2 = \left\{1 - \frac{(1-|\Gamma_L|^2)(|1-\Gamma_S s_{11}|^2 - |s_{22}-\Delta\Gamma_S|^2)}{|1-\Gamma_S s_{11} - \Gamma_L s_{22} + \Gamma_L\Gamma_S\Delta|^2}\right\} \tag{3b}$$

where s_{ij} are the elements of the scattering matrix S and Δ is the determinant of S.

Let us now consider the expression for the transducer gain G_T of the overall amplifier:

$$G_T = |s_{21}|^2 \frac{(1-|\Gamma_s|)^2(1-|\Gamma_L|^2)}{|1-\Gamma_S s_{11} - \Gamma_L s_{22} + \Gamma_L\Gamma_S\Delta|^2}. \tag{4}$$

By means of some mathematical manipulations we may introduce (3a) and (3b) into (4), obtaining the following formulation for G_T:

$$G_T = (1-|\Gamma_{\text{in}}|^2)(1-|\Gamma_{\text{out}}|^2)F \tag{5}$$

where F is given by (6), shown at the bottom of the page. Because of the unconditional stability of Q, the factor F can be put into the following useful form:

$$F = G_{T(\text{MAX})}D \tag{7}$$

where $G_{T(\text{MAX})}$ represents the maximum transducer gain (under simultaneous matched condition at input and output) of the two-port Q; it is given by

$$G_{T(\text{MAX})} = \left|\frac{s_{21}}{s_{12}}\right|(K - \sqrt{K^2-1}) \tag{8}$$

where K represents the stability factor of Q :

$$K = \frac{1-|s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|}. \tag{9}$$

The parameter D in (7) has the expression (10), which is shown at the bottom of the page. It can be easily demonstrated that in the case where the network Q is unilateral ($s_{12} = 0$), D does not depend on Γ_S and Γ_L and is equal to 1; this is also the case of perfect input and output matching ($\Gamma_{\text{in}} = \Gamma_{\text{out}} = 0$). We have found, however, that, even in the general case of $s_{12} \neq 0$, the deviation of D from unity is very small, at least when the required $|\Gamma_{\text{in}}|$ and $|\Gamma_{\text{out}}|$ are not too high (less than about 0.3). In order to verify this assumption, a numerical simulation has been performed in the case where the two-port Q is a single active device. In order to realize this simulation, Γ_S and Γ_L should be computed, given $|\Gamma_{\text{in}}|$ and $|\Gamma_{\text{out}}|$; this is not, however, possible because the phases of the two reflection coefficients are not specified. We have then proceeded as follows: for the selected active device, the scattering parameters are known for a given frequency, then (3a) and (3b) have been used to compute $|\Gamma_{\text{in}}|$ and $|\Gamma_{\text{out}}|$ for a large number of admissible values of Γ_S and Γ_L (that is, selecting magnitude less than 1 and the arbitrary phase). Then we have used (10) to compute the parameter D, using only those values of Γ_S and Γ_L for which $|\Gamma_{\text{in}}|$ and $|\Gamma_{\text{out}}|$ are less than 0.3; finally, the deviation of D from unity has been graphically represented versus $|\Gamma_{\text{in}}|$ and $|\Gamma_{\text{out}}|$ (for a given pair $|\Gamma_{\text{in}}|, |\Gamma_{\text{out}}|$ the values of Γ_S and Γ_L used in (10) are those which determine the maximum deviation of D from unity). An example of such a three-dimensional representation is shown in Fig. 3 (the active device is the GaAs FET AT10600 of Avantek, at the operating frequency of 14 GHz); it can be observed that the maximum deviation is below 0.12 for $|\Gamma_{\text{in}}|, |\Gamma_{\text{out}}| < 0.3$.

Similar results have been obtained with many other active devices at microwave frequencies. In the case of multistage amplifiers, with more than one active device in the network Q, the deviation of D from unity is even smaller, because the unilaterality of Q generally increases. This is shown in Fig. 4, where the values of D for $|\Gamma_{\text{in}}| = |\Gamma_{\text{out}}|$ have been reported at different K values (K increases with the unilaterality of Q; the figure refers to the FET AT10600 at different frequencies).

In conclusion, assuming $D = 1$, the transducer gain of a multistage amplifier can be expressed with an acceptable accuracy (as far as $|\Gamma_{\text{in}}|$ and $|\Gamma_{\text{out}}|$ are smaller than about 0.3) by the simple formula:

$$G_T \cong (1-|\Gamma_{\text{in}}|^2)(1-|\Gamma_{\text{out}}|^2)G_{T(\text{MAX})}. \tag{11}$$

III. The Design Approach

The design of multistage microwave amplifiers in a defined frequency band, based on the actual scattering pa-

$$F = |s_{21}|^2 \frac{|1-\Gamma_L s_{22} - \Gamma_S s_{11} + \Gamma_L\Gamma_S\Delta|^2}{(|1-\Gamma_L s_{22}|^2 - |s_{11}-\Delta\Gamma_L|^2)(|1-\Gamma_S s_{11}|^2 - |s_{22}-\Delta\Gamma_S|^2)}. \tag{6}$$

$$D = \frac{|s_{21}s_{12}|}{K-\sqrt{K^2-1}} \cdot \frac{|1-\Gamma_S s_{11} - \Gamma_L s_{22} + \Gamma_S\Gamma_L\Delta|^2}{(|1-\Gamma_L s_{22}|^2 - |s_{11}-\Delta\Gamma_L|^2)(|1-\Gamma_s s_{11}|^2 - |s_{22}-\Delta\Gamma_s|^2)}. \tag{10}$$

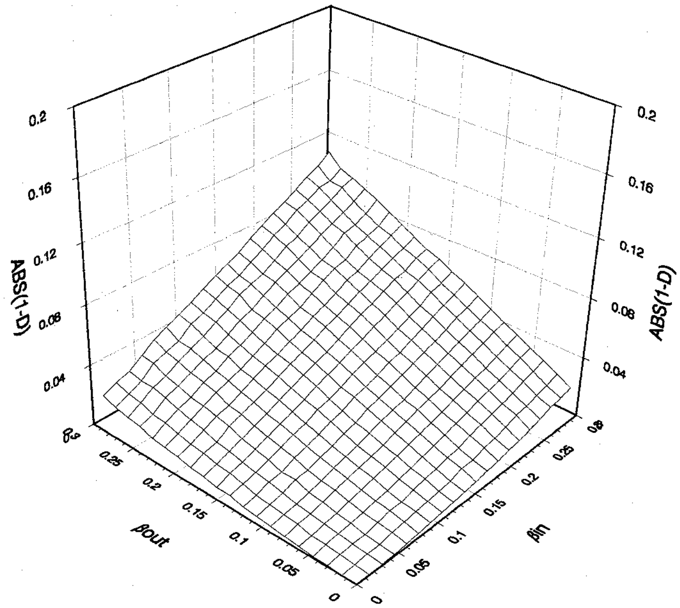


Fig. 3. Representation of the maximum deviation of D from unity as a function of the reflection coefficient at the input (β_{in}) and the output (β_{out}).

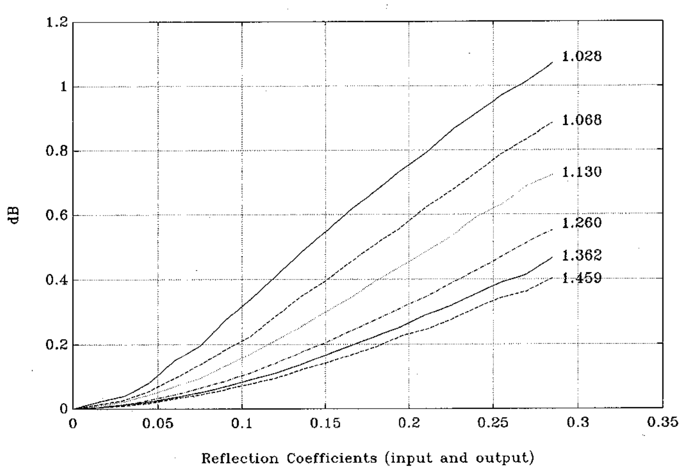


Fig. 4. Computed values of D versus $b_{\text{in}} = b_{\text{out}}$. Each curve refers to a different value of the stability factor K

rameters of the active devices employed, is generally approached by means of numerical methods: very often, in case of distributed matching networks, direct optimization is employed, especially if efficient CAD tools are available (such as TOUCHSTONE©, SUPERCOMPACT©,[1] etc.). Other approaches are based on the optimization of the coefficients of the transfer function of the matching networks (for example, the "real frequency" technique); in this case the procedure give generally better results with respect the direct optimization, but the synthesis of the networks must be performed in addition to the optimization procedure.

It is important to observe, using direct optimization, that the simultaneous design of all the equalizing networks does not give generally satisfactory results, even in the case of requirements on the transducer gain only; moreover, the results are very sensitive to the objective function employed (which, in turn, depends on the number of constraints imposed).

[1]TOUCHSTONE is a trademark of EEsof Inc.; SUPERCOMPACT is a trademark of Compact Software Inc.

We propose in this work a sequential procedure which allows the design of each equalizing network, one at a time, by imposing a single constraint in the objective function. The design approach is based on (11): a suitable range for $G_{T(\text{MAX})}$, which characterizes the network Q (Fig. 2), is determined, allowing one to design the interstage networks $N_1 \cdots N_N$. Imposing the matching requirements at the input and the output, once Q is defined, also the networks N_I and N_O can be designed. The definition of the admissible range for $G_{T(\text{MAX})}$ is the fundamental point of the design approach proposed: in fact, this range depends both on the flatness of G_T and on the levels of $|\Gamma_{\text{in}}|$ and $|\Gamma_{\text{out}}|$ required.

IV. Criteria for Defining the Admissible Range of $G_{T(\text{MAX})}$

The design specifications for a general purpose microwave amplifier are usually given as follows:

1. Frequency range: $f_1 - f_2$.
2. Transducer gain: $G_1 \leq G_T \leq G_2$.
3. Input and output match: $|\Gamma_{\text{in}}| \leq \Gamma_{iM}, |\Gamma_{\text{out}}| \leq \Gamma_{oM}$.

Let us call $G_{T\text{MAX}1}$ and $G_{T\text{MAX}2}$ the minimum and maximum values which $G_{T(\text{MAX})}$ assumes in the band of the amplifier; from (11), the following equations hold:

$$\begin{aligned} &G_1 < (1 - |\Gamma_{iM}|^2)(1 - |\Gamma_{oM}|^2) G_{T\text{MAX}1} \\ &G_2 > G_{T\text{MAX}2} \\ &G_1 < (1 - |\Gamma_{iM}|^2)(1 - |\Gamma_{oM}|^2) G_{T\text{MAX}2}. \\ &G_2 > G_{T\text{MAX}2} \end{aligned} \tag{12}$$

So the following constraint can be derived for $G_{T\text{MAX}1}$ and $G_{T\text{MAX}2}$:

$$G_2 > G_{T\text{MAX}1}, G_{T\text{MAX}2} > \frac{G_1}{(1 - |\Gamma_{iM}|^2)(1 - |\Gamma_{oM}|^2)}. \tag{13}$$

Equation (13) shows that also the requirements on G_T, $|\Gamma_{\text{in}}|$ and $|\Gamma_{\text{out}}|$ have to satisfy a constraint:

$$\frac{G_1}{G_2} < (1 - |\Gamma_{iM}|^2)(1 - |\Gamma_{oM}|^2). \tag{14}$$

Finally, being $G_2 > G_1$, the range of the allowed values of $G_{T(\text{MAX})}$, which is obtained from the given specifications, is given by

$$\frac{G_1}{(1 - |\Gamma_{oM}|^2)(1 - |\Gamma_{iM}|^2)} \leq G_{T(\text{MAX})} \leq G_2, \quad f \in \forall (f_1, f_2). \tag{15}$$

As said previously, the range of the admissible values of $G_{T(\text{MAX})}$ given by (15) refers to the network Q (Fig. 2); in the case this network is constituted by two active devices with an interstage equalizer ($N = 1$ in Fig. 1), (15) allows directly the design (in fact, $G_{T(\text{MAX})}$ is computed by means of (8) and (9), where the S parameters refer to the whole two-port Q and contain the unknown variables of the equalizer to be optimized).

The networks N_I and N_O can be designed by matching the input and the output of Q at the specifications level; this can

TABLE I
SCATTERING PARAMETERS OF GaAs MESFET AT10600

	S_{11}		S_{21}		S_{12}		S_{22}	
f_{GHz}	Mag	Phase (deg)	Mag	Phase (deg)	Mag	Phase (deg)	Mag	Phase (deg)
10	0.58	−134	2.42	81	0.11	48	0.50	−15
11	0.57	−153	2.36	70	0.11	43	0.43	−18
12	0.57	−174	2.21	57	0.11	36	0.36	−23

be performed first by computing the reflection coefficients Γ_i^0 and Γ_o^0 for the simultaneous match at input and output of Q and then by designing the two networks separately through the requirement of the match at input of the two networks (at the prescribed level), when the output is loaded with Γ_i^0 (input equalizer) or Γ_o^0 (output equalizer).

In the case of more than one interstage equalizer in the network Q (that is, for $N > 1$ in Fig. (1)) a sequential design of the networks may be realized; assuming that each active device contributes to $G_{T(\mathrm{MAX})}$ in the same amount, the interstage networks N_i are designed in sequence, starting with $i = 1$, by imposing the following constraint to the block Q_i:

$$\frac{(i+1)}{(N+1)}\frac{G_1}{(1-|\Gamma_{iM}|^2)(1-|\Gamma_{oM}|^2)} \le G_{T(\mathrm{MAX})}^i \le \frac{(i+1)}{(N+1)}G_2, \qquad \forall f \in (f_1, f_2) \tag{16}$$

the network Q_1 is built up by including up to the $(i+1)$th active device and up to the ith equalizer; at each cycle of the design sequence, only the interstage network N_i has to be designed because those with $j < i$ are known (have been obtained in the previous cycles). N_I and N_O are obtained as in the case $N = 1$.

V. Design Example

A two-stage amplifier has been designed to verify the proposed design approach; the active devices are two GaAs MESFET AT-10600, whose scattering parameters are given in Table I. A 9% relative bandwidth at a center frequency of 11 GHz has been assumed for this prototype ($f_1 = 10.5$ GHz, $f_2 = 11.5$ GHz); it should be observed that the achievable bandwidth does not depend on the approach proposed; in fact, the actual algorithm by means of which the equalizers are synthesized determines this parameter. Using numerical optimization or seminumerical techniques (for example, the "real frequency" method) in not possible to know a priori what is the maximum achievable bandwidth; if the specification for a given topology of the equalizers are not satisfied, the only way to proceed is to increase the number of components in the networks and restart the design procedure.

The overall transducer gain of the amplifier must be lower than the sum of the $G_{T(\mathrm{MAX})}$ of the transistors at the highest frequency of the band; in this case each FET has a maximum transducer gain of 10.5 dB at 11.5 GHz. The value selected for the overall G_T is then 20 ± 0.15 dB (the small margin of 1 dB with respect to the absolute maximum has been taken in order to make easier the design).

Equation (14) gives the matching requirements at input and output (assumed equals) compatible with the transducer gain

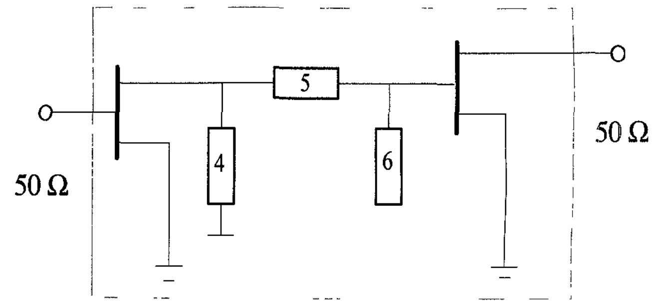


Fig. 5. Circuit for designing the interstage network.

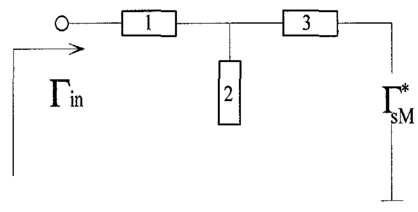


Fig. 6. Circuit for designing the input equalizer.

fluctuation:

$$|\Gamma|^2 \le 1 - \sqrt{G_1/G_2} = 0.066746.$$

It has then imposed $\Gamma_{iM} = \Gamma_{oM} = 0.15$.

The equalizing networks are assumed to be composed by ideal transmission line sections and stubs (open and/or short circuited); the design has been carried out through direct optimization, using TOUCHSTONE© program. The first step of the procedure consists in the design of the interstage network by imposing the constraint (15) on $G_{T(\mathrm{MAX})}$; in this phase the circuit actually optimized (shown in Fig. 5) is composed by the two active devices with the interstage network (the input and output equalizers do not affect $G_{T(\mathrm{MAX})}$ as far as dissipative components are not included). Note that once an interstage network is obtained which satisfy the constraint (15) all over the amplifier bandwidth, the two-port Q (Fig. 2), which is unconditionally stable, is also defined; it is then possible to compute the reflection coefficients Γ_{sM} and Γ_{1M} for the simultaneous match at input and output of the amplifier (these values depend on the frequency and can be obtained directly as an output measurements from TOUCHSTONE©). The input and output equalizers can be designed separately by imposing the transformation of Γ_{sM} and Γ_{1M} to 50 Ω (Figs. 6 and 7); considering the input equalizer the design is performed by loading the output of this network with $(\Gamma_{sM})^*$ and imposing, through optimizing, that the reflection coefficient at the input be less than 0.15. In a similar way also the output equalizer is obtained.

Fig. 8 reports the overall amplifier structure; the values obtained for the characteristic impedance and the electrical length of the transmission lines (at 11 GHz) are shown in Table II.

The computed performances in the band of the amplifier are reported in Figs. 9 and 10; the first shows the transducer gains G_T and $G_{T(\mathrm{MAX})}$; in the second the reflection coefficients at the input and output are given. Note that all the design requirements are well fulfilled.

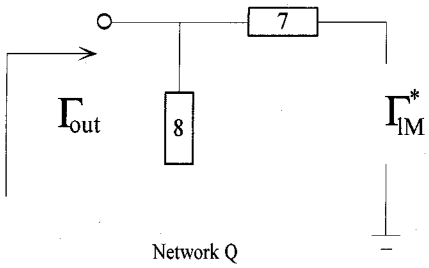


Fig. 7. Circuit for designing the output equalizer.

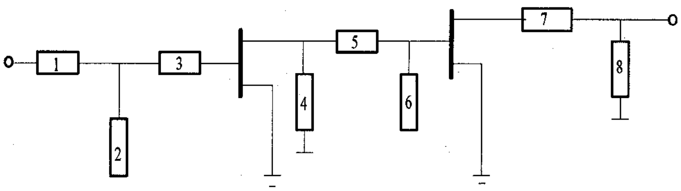


Fig. 8. Final schematic structure of the amplifier.

TABLE II
ELECTRICAL PARAMETERS OF THE OVERALL AMPLIFIER
Z_c is the characteristic impedance, E is the electrical length at 11 GHz.

Element Number	Z_c (Ω)*	E (deg)*
1	18.90	69.80
2	90.50	100.10
3	39.30	15.50
4	185.40	106.80
5	29.20	58.10
6	37.90	123.30
7	95.00	72.30
8	20.20	95.30

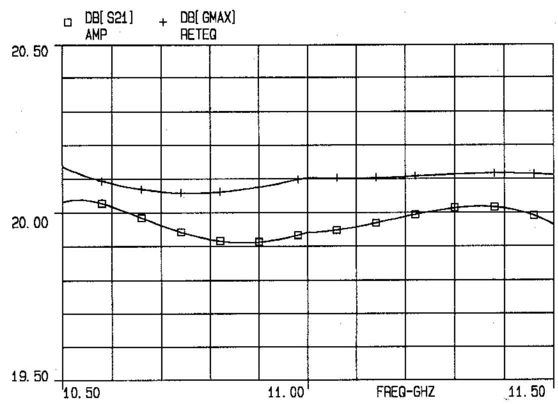


Fig. 9. Transducer gains G_T and $G_{T(\mathrm{MAX})}$ of the designed amplifier.

VI. CONCLUSIONS

A design approach to multistage amplifiers with prescribed matching levels at input and output has been presented in this paper. The method allows us to design separately the equalizing networks, assumed lossless, by requiring single constraints to be satisfied. It can be implemented either using the direct optimization with commercial software (TOUCHSTONE©, SUPERCOMPACT©), or through efficient numerical methods (the "real frequency" technique). A design example has been presented which validate the proposed design approach.

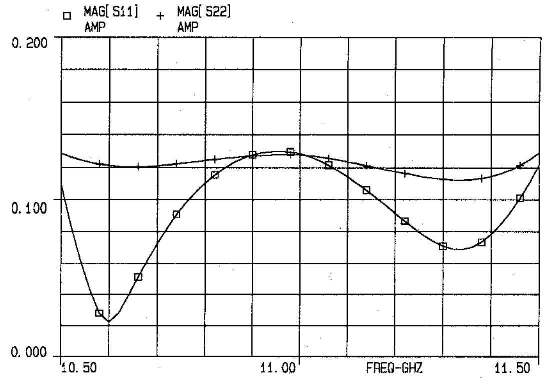
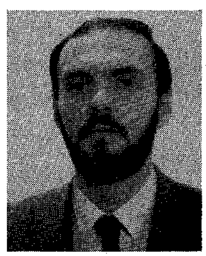


Fig. 10. Magnitude of the reflection coefficients at the input and the output of the designed amplifier.

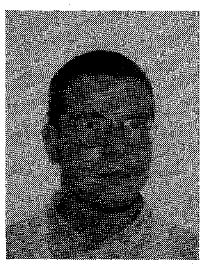
REFERENCES

[1] H. J. Carlin and J. J. Komiak, "A new method of broadband equalization applied to microwave amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, Feb. 1979.
[2] B. S. Yarman and H. J. Carlin, "A simplified real frequency technique applied to broadband multistage microwave amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, Dec. 1982.
[3] G. Capponi and B. Di Maio, "Computer aided design of multistage amplifiers," *Alta Freq.*, vol. 53, no. 4, July 1984.
[4] G. Macchiarella, M. Politi, and G. B. Stracca, "Improved CAD procedures for multistage broadband amplifiers," in *Proc. MIOP'88*, Mar. 1988.



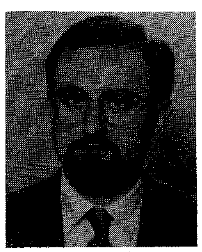
Giuseppe Macchiarella received the "Laurea" degree in electronic engineering in 1975 from the Politecnico di Milano, Italy.

From 1977 to 1987 he has been with Centro Studi per le Telecomunicazioni Spaziali of the National Research Council of Italy (CNR), where he has been involved in microwave propagation studies (SIRIO satellite experiment). Since 1987 he has been Associate Professor at the Electronic Department of Politecnico di Milano. His main interests are in the field of active and passive microwave circuits and in numerical techniques for electromagnetics.



Alessandro Raggi was born in Rome, Italy, in 1963. He received the "Laurea" degree in electronic engineering from the Politecnico di Milano in 1990.

In 1991 he joined Fabbrica Italiana Apparecchiature Radioelettriche (FIAR), Milan, where he is currently working in the field of pulsed Doppler radar.



Elpidio di Lorenzo was born in Casalnuovo (Naples, Italy) in 1963. He received the "Laurea" degree in electronic engineering from the Politecnico di Milano in 1990. During 1990–1991 he served in the Army.

Since 1992 he has been working at Italtel SIT, Transmission Business Unit—Optical Interface Laboratory, Milan, where he is engaged in monolitic circuit design of optical fiber transceivers, and particularly the design of data and clock recovery circuit.